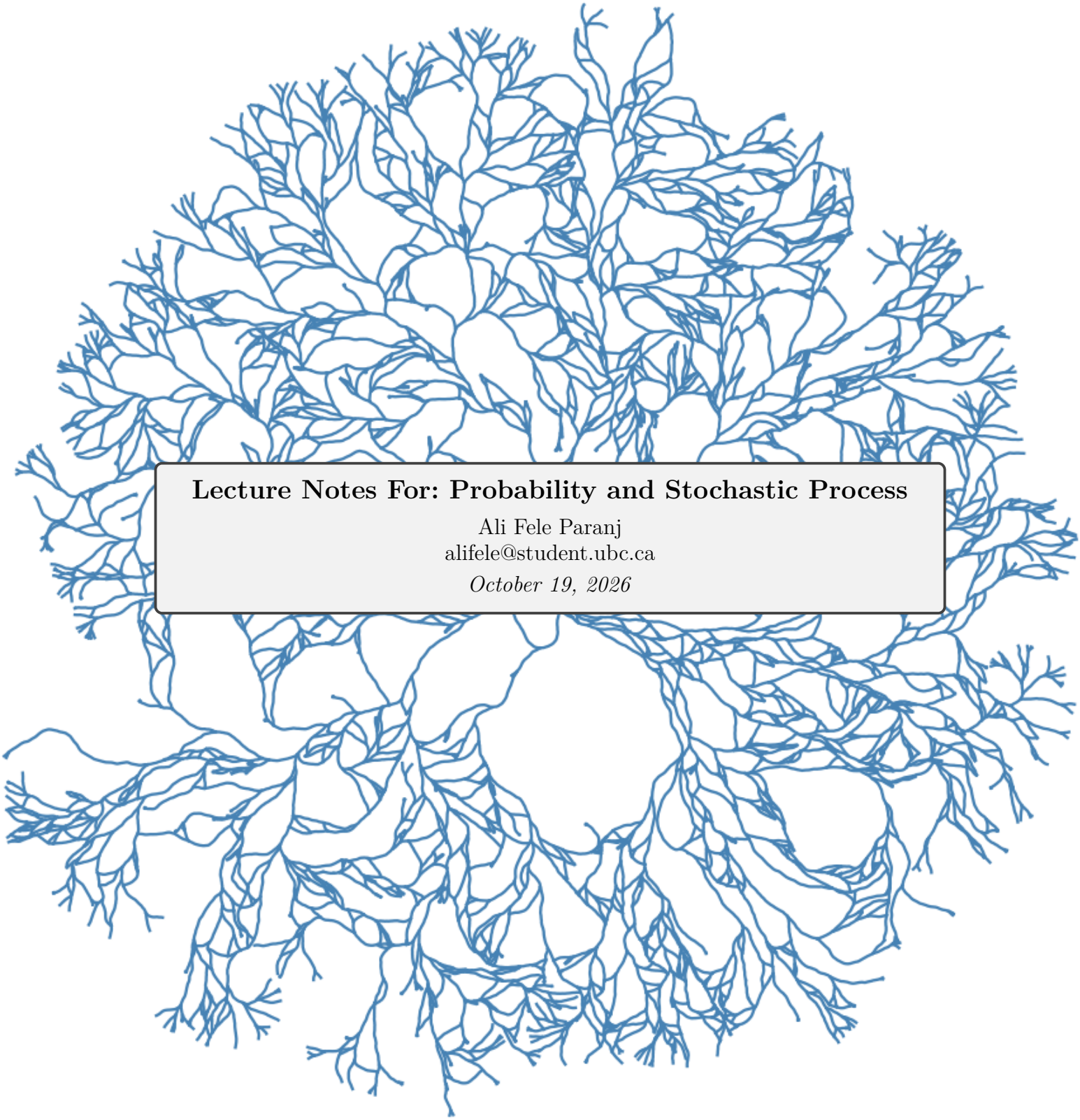


Lecture Notes For: Probability and Stochastic Process

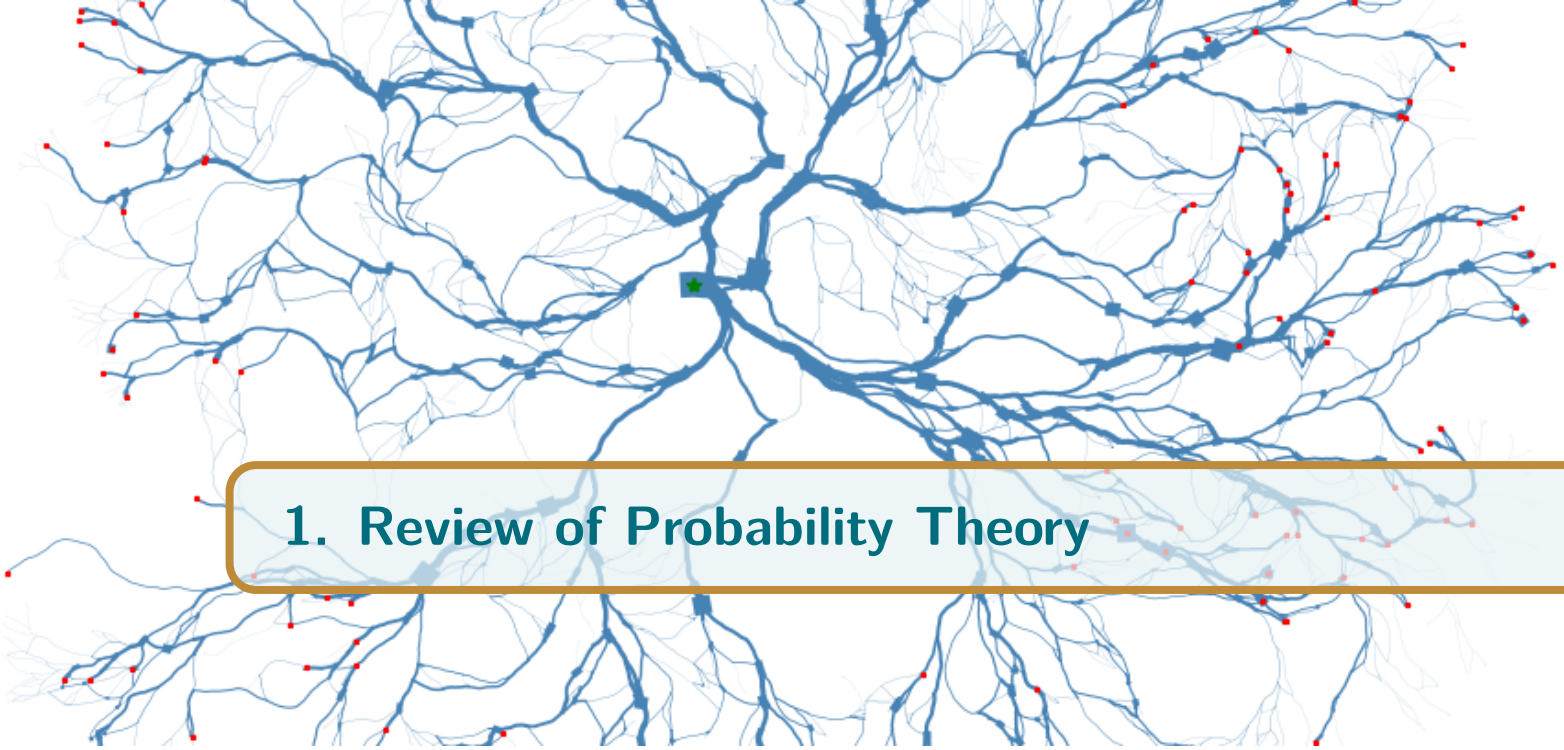
Ali Fele Paranj
alifele@student.ubc.ca

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1. Review of Probability Theory

■ **Problem 1.1** Consider any $T \subset \mathbb{R}^n$. Show that $\text{conv}(T)$ is a convex set.

Solution For the possibly infinite set, the definition of the convex hall, is the following: $\forall x \in T$, there are points $p_i \in T$ and $\exists \alpha_i \in \mathbb{R}$ such that

$$\sum_i \alpha_i p_i = x$$

such that $\sum_i \alpha_i = 1$.

So let $x, y \in \text{conv}(T)$. We want to show that $\lambda x + (1 - \lambda y) \in T$. From definition it follows that $\exists \{p_i\}_{i=1}^l$ and $\exists \{q_i\}_{i=1}^k$, and real coefficients $\alpha_i \in \mathbb{R}$ and $\beta_i \in \mathbb{R}$ such that

$$\sum_i \alpha_i p_i = x, \qquad \sum_j \beta_j q_j = y,$$

such that $\sum_i \alpha_i = 1$ and $\sum_j \beta_j = 1$. Observe that

$$\begin{aligned} \lambda x + (1-\lambda) y &= \lambda \sum_i \alpha_i p_i + (1-\lambda) \sum_j \beta_j q_j \\ &= \sum_{i,j} \lambda \alpha_i p_i + (1-\lambda) \beta_j q_j. \end{aligned}$$

Observe that $\sum_{i,j} \lambda \alpha_i + (1-\lambda)\beta_j = \lambda + (1-\lambda) = 1$. So this implies that $\lambda x + (1-\lambda)y$ is indeed in $\text{conv}(T)$.

■ **Problem 1.2** Show that the point-wise maximum of a finite number of convex functions, is a convex function.

Solution Let $\{f_i\}$ be a finite set of convex function, and define

$$f(x) := \max_i f_i(x).$$

Observe that

$$\lambda f(x) + (1-\lambda) f(y) \geq \lambda f_i(x) + (1-\lambda) f_i(y) \qquad \geq f_i(\lambda x + (1-\lambda) y)$$

Because the inequality above holds for all choices of i, it implies that

$$\lambda f(x) + (1-\lambda) f(y) \geq f(\lambda x + (1-\lambda) y).$$

■ **Problem 1.3** The definition of a convex function involves convex combinations of two points x and y. Let's extend it to arbitrary many points. Let $k \subset \mathbb{R}^n$ be a convex subset. Prove that a function $f : K \to \mathbb{R}$ in convex if and only if the following holds: For any $m \in \mathbb{N}$, and vectors $x_i \in K$ and any numbers $\lambda_i \geq 0$ with $\sum_i \lambda_i = 1$, we have

$$f\left(\sum_i \lambda_i x_i\right) \leq \sum_i^n \lambda_i f(x_i).$$